الجامعة المستنصرية
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المرحلة الثالثة / مسائي

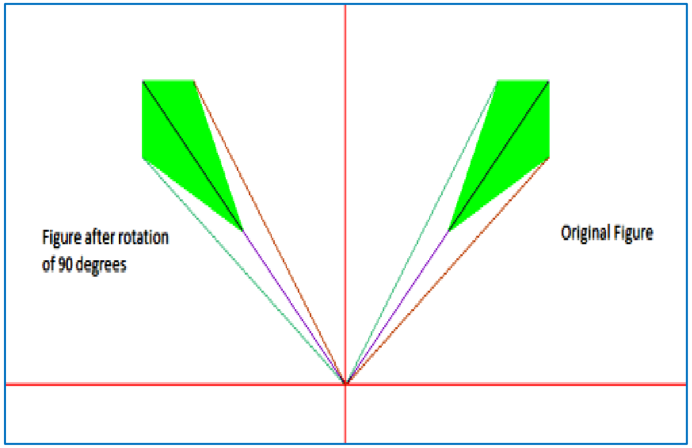
Computer Graphics

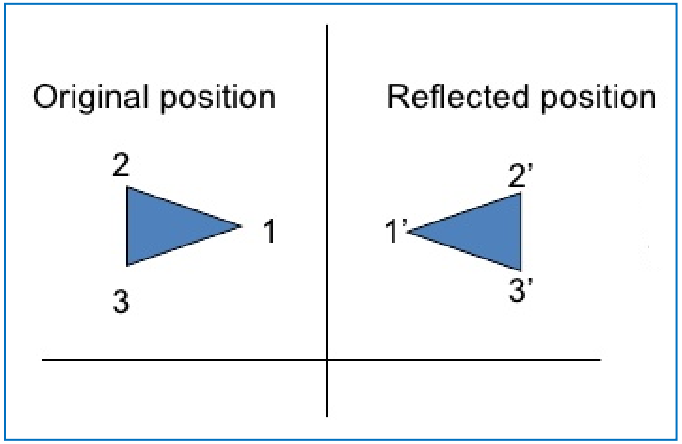
PART 7

Part Seven

1. ***Rotation***
   - Rotation about the origin
   - Rotate about a specific point (XP, YP)

2. **Reflection**
   - Reflection on an arbitrary line





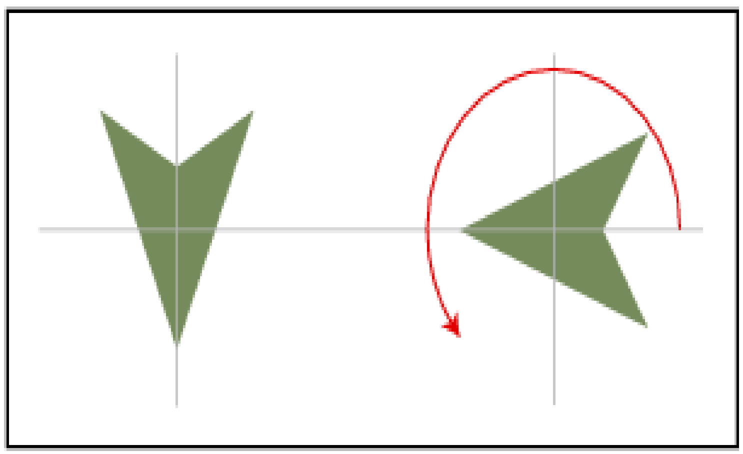
Rotation

Another useful transformation is the rotation of an object about specified pivot point. In rotation, the object is rotated Ó about the origin.

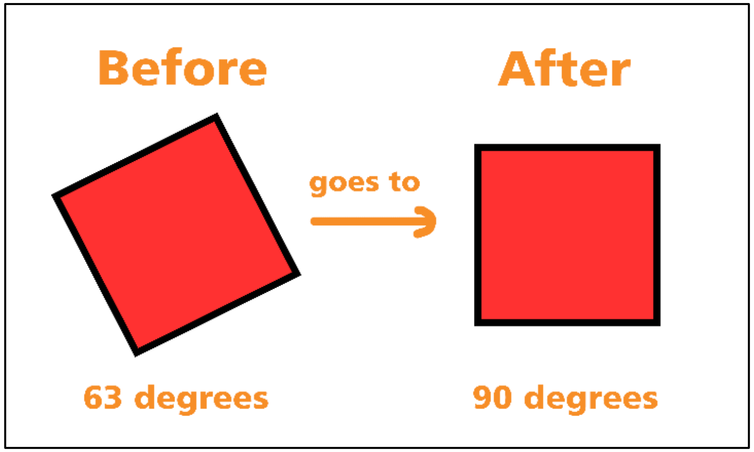
The convention is that :

1- The direction of rotation is counterclockwise if Ó is a positive angle

2- The direction of rotation is clockwise if Ó is a negative angle.



(a) counterclockwise if Ó is a positive angle



(b) clockwise if Ó is a negative angle

Rotation about the origin

The rotation matrix to rotate an object about the origin in an anticlockwise direction is:

Cos θ	Sin θ	0
-Sin θ	Cos θ	0
0	0	1

Alternatively, in the equation:

$$X_{new} = X * \text{Cos}\,\theta - Y * \text{Sin}\,\theta$$

$$Y_{new} = Y * \text{Cos}\,\theta + X * \text{Sin}\,\theta$$

The form of the rotation matrix to rotate an object about the origin in an anticlockwise direction :

Cos θ	Sin θ	0
-Sin θ	Cos θ	0
0	0	1

When Ó =90

Sin (90) = 1

Cos (90) = 0

0	1	0
-1	0	0
0	0	1

When Ó =180

Sin (180) = 0

Cos (180) = -1

-1	0	0
0	-1	0
0	0	1

When Ó =270

Sin (270) = -1

Cos (270) = 0

0	-1	0
1	0	0
0	0	1

When Ó =360

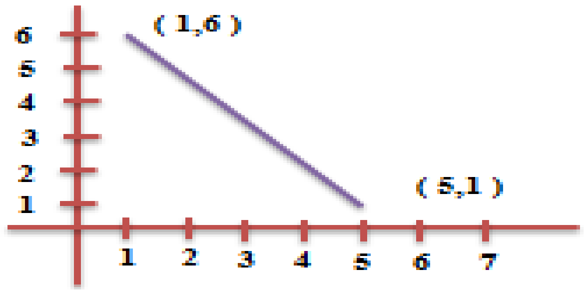
Sin (360) = 0

Cos (360) = 1

1	0	0
0	1	0
0	0	1

Example 1:

Rotate the line P1 (1, 6) and P2 (5, 1) anticlockwise 90 degree.



Original line

The solve 1:

$$X_{new} = X * \text{Cos}\,\theta - Y * \text{Sin}\,\theta$$

$$Y_{new} = Y * \text{Cos}\,\theta + X * \text{Sin}\,\theta$$

First point

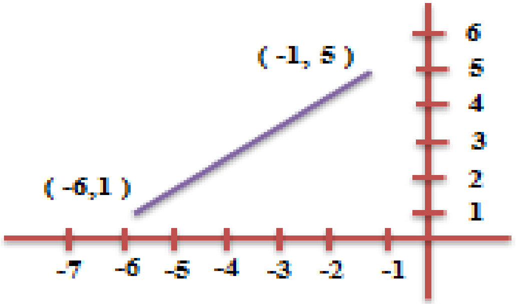
X_{new} = 1*0 – 6* 1 = -6

y_{new} = 6*0 + 1*1 = 1

Second point

X_{new} = 5*0 – 1* 1 = -1

y_{new} = 1*0 + 5*1 = 5

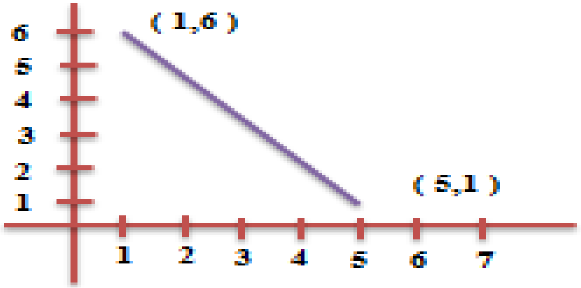


Line after rotation

anticlockwise if Ó is a positive angle

Example 1:

Rotate the line P1 (1, 6) and P2 (5, 1) anticlockwise 90 degree.



Original line

The solve 2:

X1 new	Y1 new	1
X2 new	Y2 new	1

=

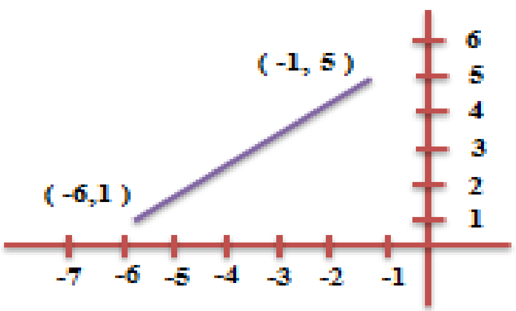
1	6	1
5	1	1

*

0	1	0
-1	0	0
0	0	1

=

-6	1	1
-1	5	1



Line after rotation

anticlockwise if Ø is a positive angle

Rotation in clockwise direction:

In order to rotate in clockwise direction we use a negative angle, and because:

Cos (-Ǿ) = Cos Ǿ

Sin (-Ǿ) = - Sin Ǿ

Therefore, The form of the rotation matrix to rotate an object about the origin in clockwise direction :

Cos Ǿ	-Sin Ǿ	0
Sin Ǿ	Cos Ǿ	0
0	0	1

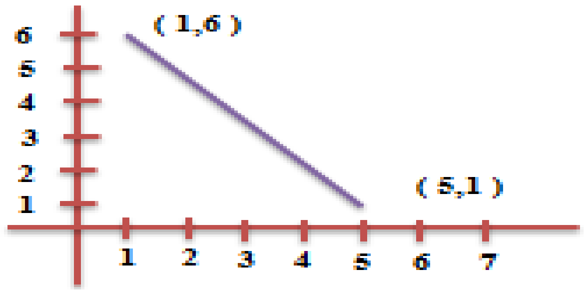
Alternatively, in the equation:

Xnew =X * Cos Ǿ + Y * Sin Ǿ

Ynew = Y * Cos Ǿ - X * Sin Ǿ

Example 2:

Rotate the line P1 (1, 6) and P2 (5, 1) in clockwise (-90) degree.



Original line

The solve:

X1 new	Y1 new	1
X2 new	Y2 new	1

=

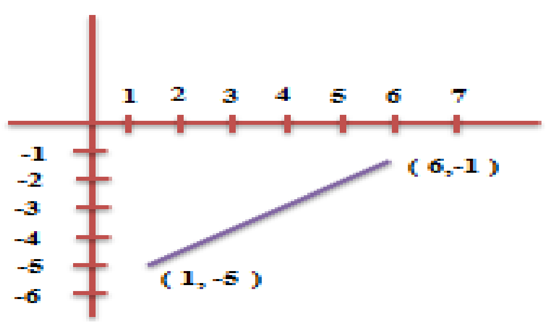
1	6	1
5	1	1

*

0	-1	0
1	0	0
0	0	1

=

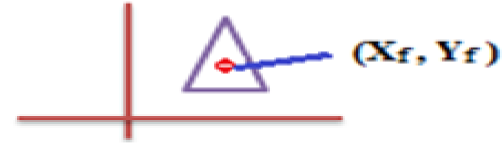
6	-1	1
1	-5	1



Line after rotation

clockwise if Ø is a negative angle

Rotate about a specific point (XP, YP)



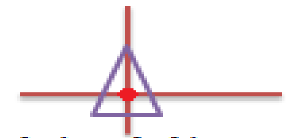
original position of object and pivot point

We need three steps:

1- Translate the points (and the object) so that the point (XP,YP) lies on the origin

XP1= X - XP

YP1= Y – YP

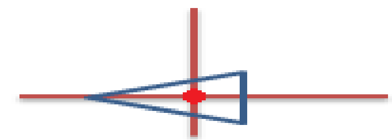


translation of object so that pivot point (X_f, Y_f) is at origin

2- Rotate the translated point (and the translated object) by Ó degree about the origin to obtain the new point (XP2,YP2)

XP2=XP1 * Cos Ó - YP1 * Sin Ó

YP2=YP1 * Cos Ó + XP1 * Sin Ó

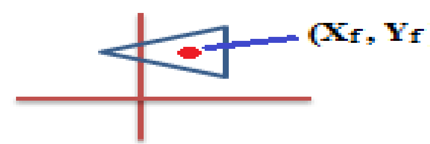


rotation about origin

3- Back translation

XP3= XP2 + XP

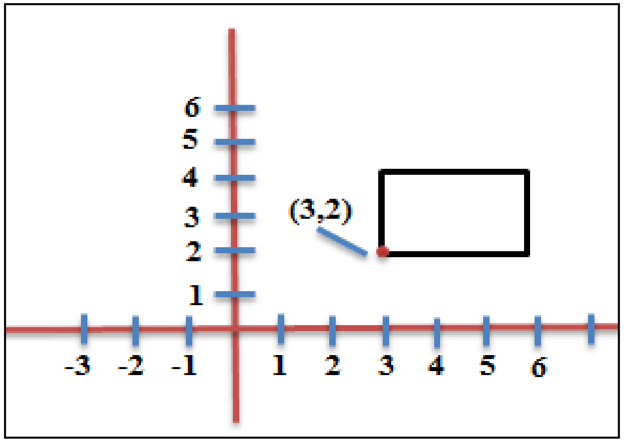
YP3= YP2 + YP



translation of object so that pivot point is returned to position (X_f, Y_f)

Example 3:

Rotate the rectangle (3, 2),(6, 2),(3, 4),(6, 4) counterclockwise with Ǿ =90 around the point (3,2).



The solve:

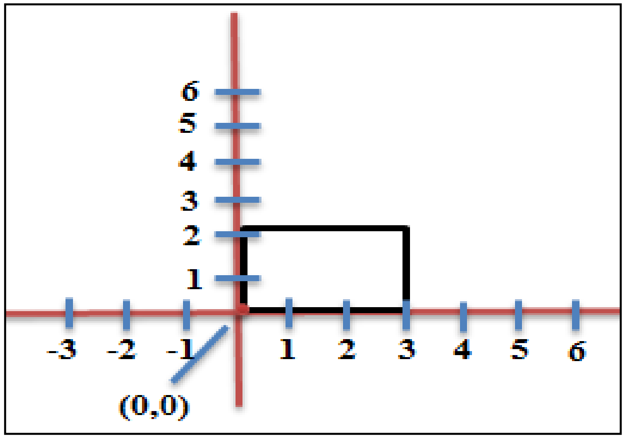
1- Translation

XP= 3 , YP= 2

XP1= X - XP

YP1= Y – YP

Point (3,2) $X1_{new} = 3 - 3 = 0$ $Y1_{new} = 2 - 2 = 0$	**Point (6,2)** $X2_{new} = 6 - 3 = 3$ $Y2_{new} = 2 - 2 = 0$
Point (3,4) $X3_{new} = 3 - 3 = 0$ $Y3_{new} = 4 - 2 = 2$	**Point (6,4)** $X4_{new} = 6 - 3 = 3$ $Y4_{new} = 4 - 2 = 2$



2- Rotation

When Ó =90

Sin (90) = 1

Cos (90) = 0

XP2=XP1 * Cos Ó - YP1 * Sin Ó

YP2=YP1 * Cos Ó + XP1 * Sin Ó

0	1	0
-1	0	0
0	0	1

Point (0,0)

$X1_{new} = 0 * (0) – 0 * (1) = 0$

$Y1_{new} = 0 * (0) + 0 * (1) = 0$

Point (0,2)

$X3_{new} = 0 * (0) – 2 * (1) = -2$

$Y3_{new} = 2 * (0) + 0 * (1) = 0$

Point (3,0)

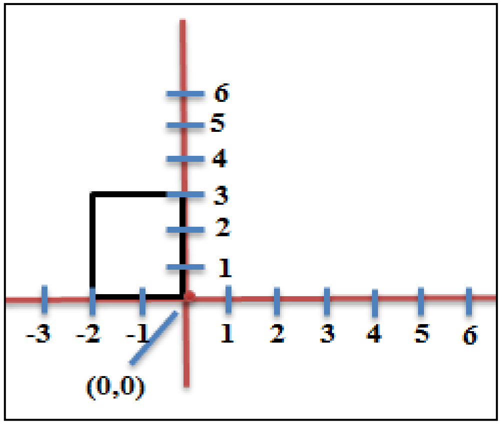
$X2_{new} = 3 * (0) – 0 * (1) = 0$

$Y2_{new} = 0 * (0) + 3 * (1) = 3$

Point (3,2)

$X4_{new} = 3 * (0) – 2 * (1) = -2$

$Y4_{new} = 2 * (0) + 3 * (1) = 3$



3- Back Translation

$XP3 = XP2 + XP$

$YP3 = YP2 + YP$

Point (0,0)

$X1_{new} = 0 + 3 = 3$

$Y1_{new} = 0 + 2 = 2$

Point (-2,0)

$X3_{new} = -2 + 3 = 1$

$Y3_{new} = 0 + 2 = 2$

Point (0,3)

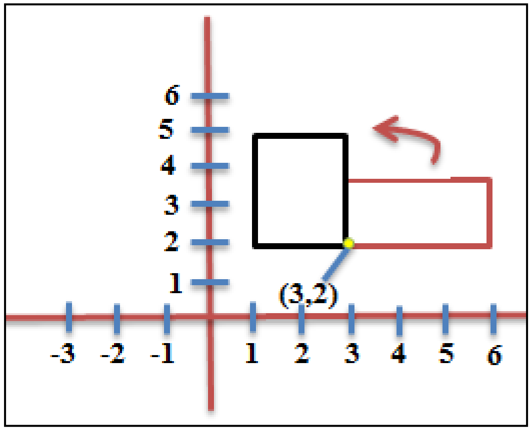
$X2_{new} = 0 + 3 = 3$

$Y2_{new} = 3 + 2 = 5$

Point (-2,3)

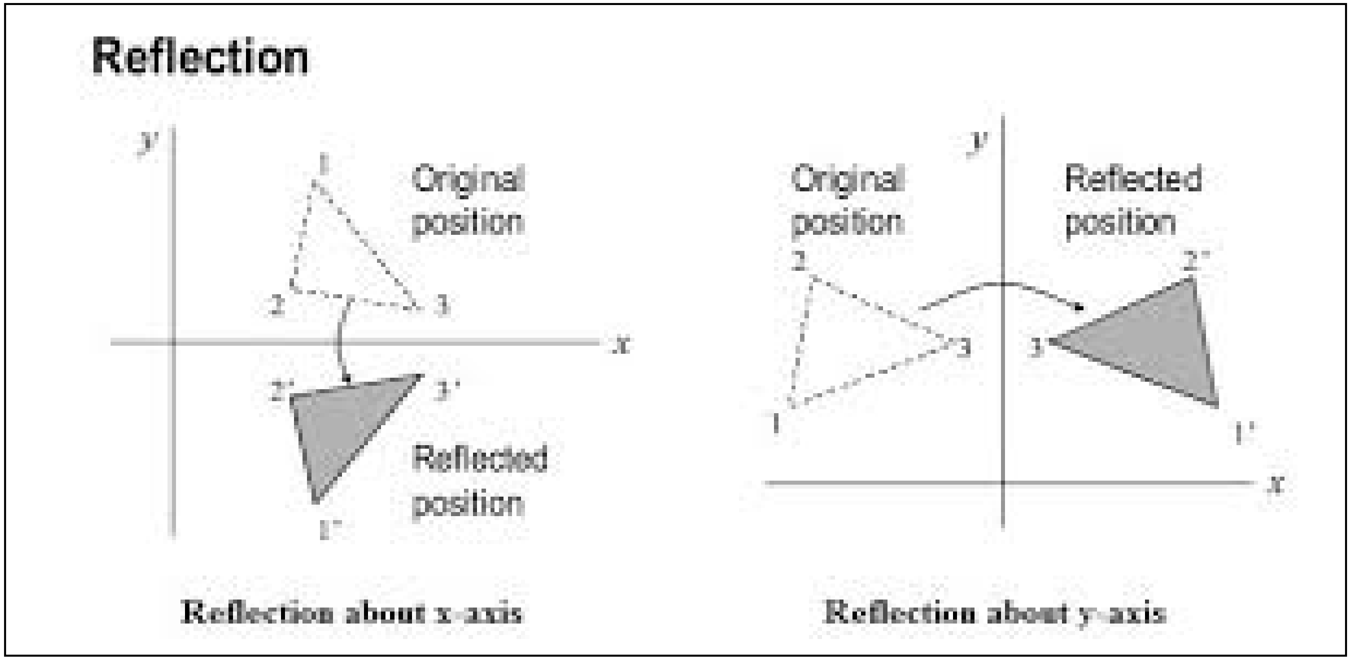
$X4_{new} = -2 + 3 = 1$

$Y4_{new} = 3 + 2 = 5$



Reflection

A reflection is a transformation that produces a mirror image of an object relative to an axis of reflection. We can choose an axis of reflection in the x-y plane or perpendicular to the x-y plane. The figure below gives an example of the reflection in the y-direction and in the x-direction.

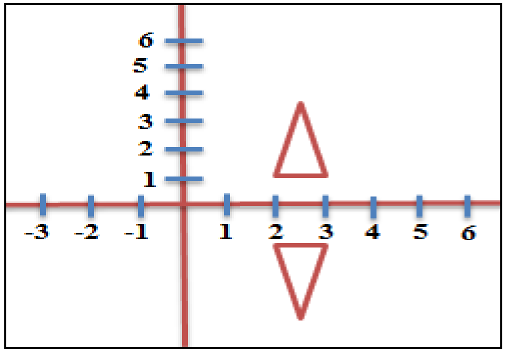


1- Reflection on the X-axis

$X_{new} = X$
$Y_{new} = -Y$

OR

1	0	0
0	-1	0
0	0	1

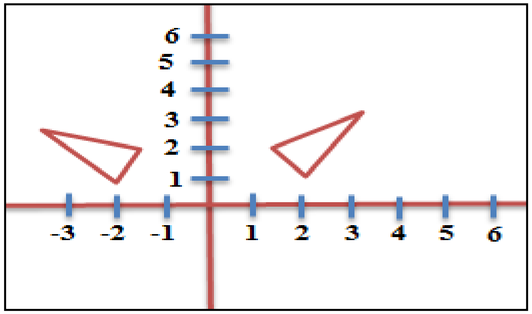


2- Reflection on the Y-axis

$X_{new} = -X$
$Y_{new} = Y$

OR

-1	0	0
0	1	0
0	0	1

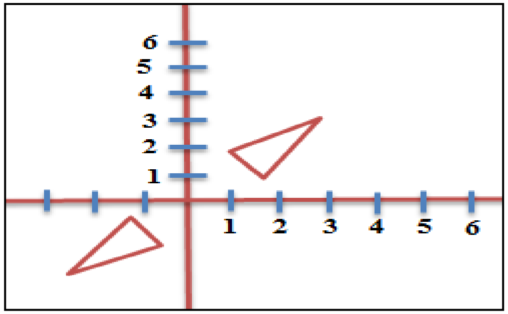


3- Reflection on the origin

$X_{new} = -X$
$Y_{new} = -Y$

OR

-1	0	0
0	-1	0
0	0	1

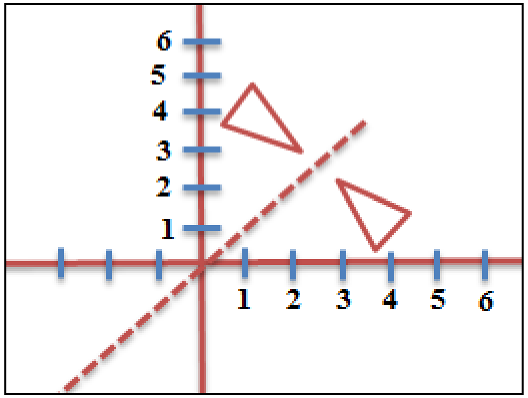


4- Reflection on the line Y = X

$X_{new} = Y$
$Y_{new} = X$

OR

0	1	0
1	0	0
0	0	1

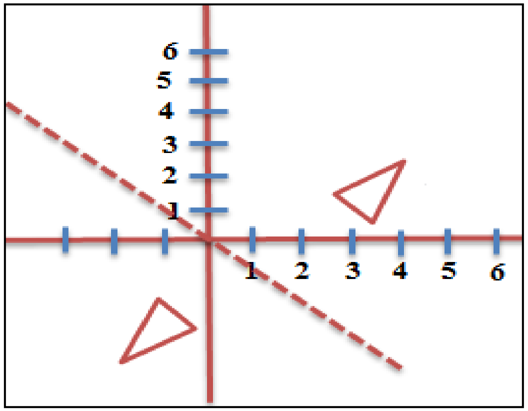


5- Reflection on the line Y= -X

$X_{new} = -Y$
$Y_{new} = -X$

OR

0	-1	0
-1	0	0
0	0	1



Example 1:

Reflect the point P(3, 2) in : A- X axis; B- Y axis; C- origin; D-line Y=X;

The solve:

A- X axis

X1 new	Y1 new	1

=

3	2	1

*

1	0	0
0	-1	0
0	0	1

=

3	-2	1

B- Y axis

X1 new	Y1 new	1

=

3	2	1

*

-1	0	0
0	1	0
0	0	1

=

-3	2	1

C- Origin

X1 new	Y1 new	1

=

3	2	1

*

-1	0	0
0	-1	0
0	0	1

=

-3	-2	1

D- Line Y=X

X1 new	Y1 new	1

=

3	2	1

*

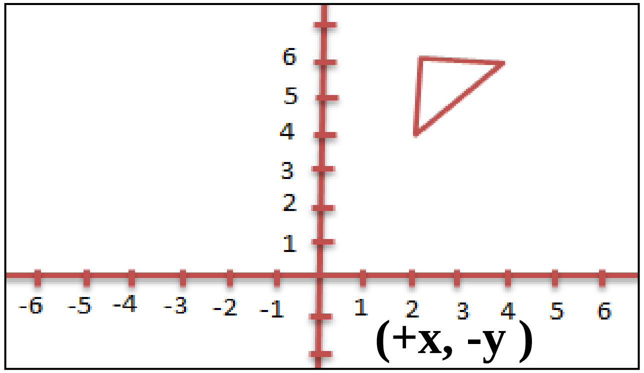
0	1	0
1	0	0
0	0	1

=

2	3	1

Example 2 :

Reflect the triangle with vertices at A(2, 4), B(4, 6), C(2, 6) in : A- X axis



1- The solve:

$X_{new} = X$

$Y_{new} = -Y$

Point (2,4)

$X1_{new} = 2$

$Y1_{new} = -4$

Point (4,6)

$X1_{new} = 4$

$Y1_{new} = -6$

Point (2,6)

$X1_{new} = 2$

$Y1_{new} = -6$

2- The solve:

X1 new	Y1 new	1
X2 new	Y2 new	1
X3 new	Y3 new	1

=

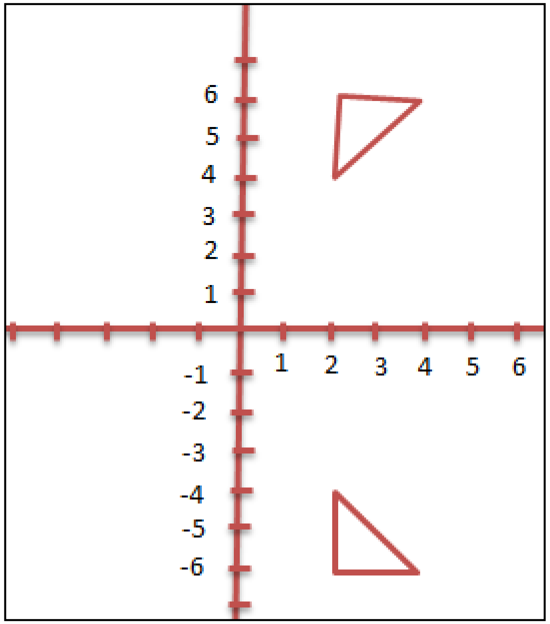
2	4	1
4	6	1
2	6	1

*

1	0	0
0	-1	0
0	0	1

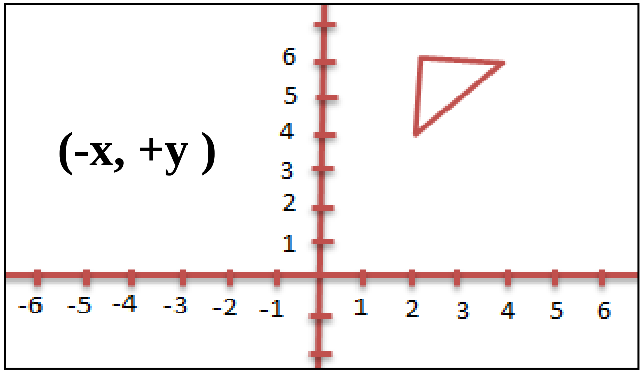
=

2	-4	1
4	-6	1
2	-6	1



Example 3 :

Reflect the triangle with vertices at A(2, 4), B(4, 6), C(2, 6) in : Y axis;



1- The solve

$X_{new} = - X$

$Y_{new} = Y$

Point (2,4)

$X1_{new} = -2$

$Y1_{new} = 4$

Point (4,6)

$X1_{new} =- 4$

$Y1_{new} = 6$

Point (2,6)

$X1_{new} = -2$

$Y1_{new} = 6$

2- The solve

X1 new	Y1 new	1
X2 new	Y2 new	1
X3 new	Y3 new	1

=

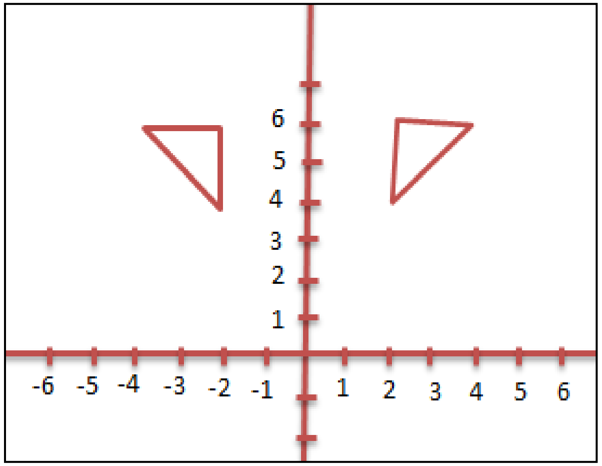
2	4	1
4	6	1
2	6	1

*

-1	0	0
0	1	0
0	0	1

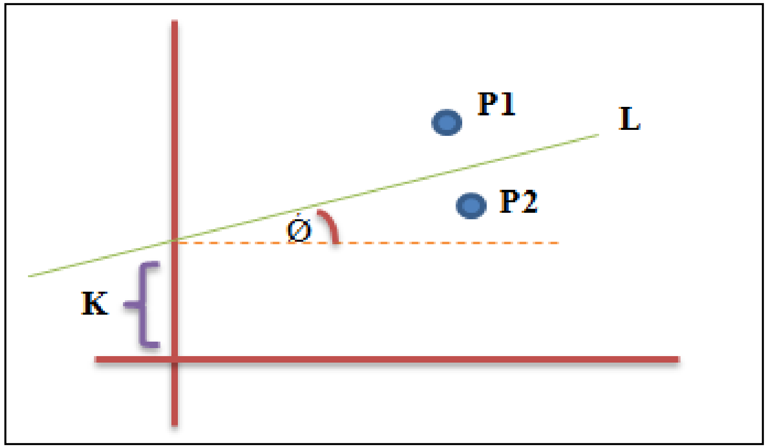
=

-2	4	1
-4	6	1
-2	6	1



Reflection on an arbitrary line

To reflect an object on a line that does not pass through the origin, which is the general case:



As shown in the figure, let the line L intercept with Y axis in the point (0,K) and have an angle of inclination Ó degree with respect to the positive direction of X axis .

To reflect the point P1 on the line L, we follow the following steps:

1. Move all the points up or down (in the direction of Y axis) so that L pass through the origin

T =

1	0	0
0	1	0
0	- k	1

2. Rotate all the points through (-Ǿ) degree about the origin making L lie along the X axis

R =

Cos θ	-Sin θ	0
Sin θ	Cos θ	0
0	0	1

3. Reflect the point P1 on the X axis

R efX=

1	0	0
0	-1	0
0	0	1

4. Rotate back the points by (-Ǿ) degree so that L back to its original orientation

R_{-1} =

Cos θ	Sin θ	0
-Sin θ	Cos θ	0
0	0	1

5. **Shift in the direction of Y axis so that L is back in its original position**

T_{-1} =

1	0	0
0	1	0
0	k	1

The sequence of matrices needed to perform this non-standard reflection is:

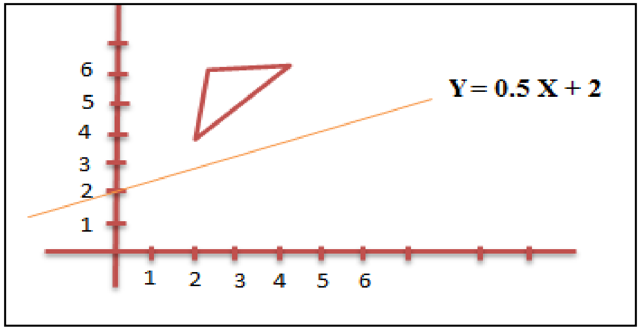
$$S=T * R * RefX * R_{-1} * T_{-1}$$

S =

Cos 2Ǿ	Sin 2Ǿ	0
Sin 2Ǿ	-Cos 2Ǿ	0
-K Sin 2Ǿ	K+K Cos 2Ǿ	1

Example 4 :

Find the single matrix that causes all the points in the plane to be reflected in the line with equation Y=0.5X+2 , then apply this matrix to reflect the triangle with vertices at A(2, 4), B(4, 6), C(2, 6) in the line.



The solve

- **The Cartesian equation of a line in 2D is *Y= M * X +b* where b is the intersection of the lint with the Y axis and M is gradient of the line *M= ΔY / ΔX = Tan Ǿ***

- **So the line *Y=0.5 X + 2* has gradient *M= 0.5* and intersect with the Y axis at the point where *y=2***

- **So K=2 , Tan Ǿ = 0.5 ==> Ǿ = 26.57 2Ǿ=53.13 , Cos 2Ǿ = 0.6 , Sin 2Ǿ = 0.8**

S =

0.6	0.8	0
0.8	- 0.6	0
- 1.6	3.2	1

To reflected the triangle on the line:

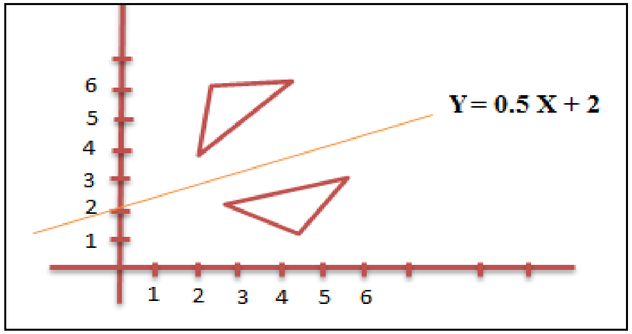
2	4	1
4	6	1
2	6	1

*

0.6	0.8	0
0.8	- 0.6	0
- 1.6	3.2	1

=

2.8	2.4	1
5.6	2.8	1
4.4	1.2	1



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