Chapter Three, Two – Dimension Transformation

Chapter Three

Two Dimension Transformation

Fundamental to all computer graphics systems is the ability to simulate the movement and the manipulation of objects in the plane. These processes are described in terms of :

1. Translation
2. Scaling
3. Rotation
4. Reflection
5. Shearing

Our object is to describe these operations in a mathematical form suitable for computer processing.

There are two complementary points of view for describing object movement:

1- Geometric transformation: the object itself is moved relative to a stationary coordinate system or background. Geometric transformation is applied to each point of the object.

2- Coordinate transformation: the object is held stationary while the coordinate system is moved relative to the object, for example, the motion of a car in a scene; we can keep the car fixed while moving the background scenery.

3.1 Geometric transformation

1- Translation

In translation, an object is displaced a given distance and direction from its original position. A point in the XY plane can be translated by adding a translation amount to the coordinates of the point. For each point P(X, Y) which is to be moved by TX units parallel to the X-axis and by TY units parallel to the Y-axis to the new point P2(X2, Y2) we use the equations:

$$X2=X+TX$$

$$Y2=Y+TY$$

If TX is positive then the point moves to the right

If TX is negative then the point moves to left

If TY is positive then the point moves up (in PC moves down)

If TY is negative then the point moves down (in PC moves up).

The transformation of Translation can be represented by (3*3) matrix:

1	0	0
0	1	0
TX	TY	1

Example 3.1

Move the line (-6,-4), (7,-5) 3 units in the X direction and 2 units in the Y direction

Solution:

TX=3, TY=2

First point

X1new= -6 + 3= -3

Y1new= -4 + 2= -2

Second point

X2new= 7 + 3= 10

Y2new= -5 + 2= - 3

By using the matrix representation

$$\begin{bmatrix} X1new & Y1new & 1 \\ X2new & Y2new & 1 \end{bmatrix} = \begin{bmatrix} -6 & -4 & 1 \\ 7 & -5 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

2- Scaling

Scaling is the process of expanding or compressing the dimensions of an object (changing the size of an object). The size of an object can be changed by multiplying the points of an object by scaling factor.

If SF (scale factor) > 1 then the object is enlarged

If SF (scale factor) < 1 then the object is compressed

If SF (scale factor) = 1 then the object is unchanged

SX is the scale factor in the X direction

SY is the scale factor in the Y direction

To scale a point P(X,Y) we use the equations:

Xnew = X * SX

Ynew = Y * SY

If SX and SY have the same value (SX=SY) then the scaling is said to be ***homogeneous*** **(or *balanced*).**

By using the matrix representation

Xnew	Ynew	1

=

X	Y	1

*

SX	0	0
0	SY	0
0	0	1

Example 3.2: Scale the rectangle (12,4),(20,4),(12,8),(20,8) with SX=2,SY=2

Solution: (By using the equations)

For the point (12,4)
X1new= 12 * SX =12 * 2 = 24
Y1new= 4 * SY = 4 * 2 = 8

For the point (20,4)
X2new= 20 * SX =20 * 2 = 40
Y2new= 4 * SY = 4 * 2 = 8

For the point (12,8)
X3new= 12 * SX =12 * 2 = 24
Y3new= 8 * SY = 8 * 2 = 16

For the point (20,8)
X4new= 20 * SX = 20 * 2 =40
Y4new= 8 * SY = 8 * 2 = 16

Solution: (by using matrices)

X1new	Y1new	1
X2new	Y2new	1
X3new	Y3new	1
X4new	Y4new	1

=

12	4	1
20	4	1
12	8	1
20	8	1

*

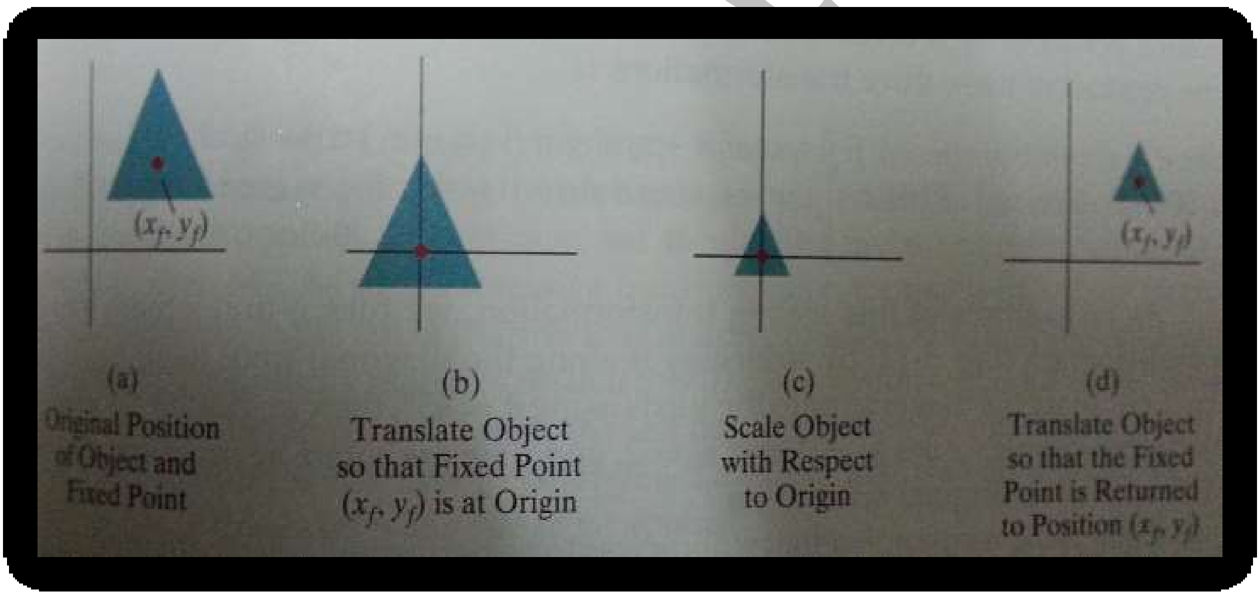
2	0	0
0	2	0
0	0	1

=

24	8	1
40	8	1
24	16	1
40	16	1

Notice that after a scaling transformation is performed, the new object is located at a different position relative to the origin. In fact, in scaling transformation, the only point that remains fixed is the origin. If we want to let one point of an object that remains at the same location (fixed), scaling can be performed by three steps:

1. Translate the fixed point to the origin, and all the points of the object must be moved the same distance and direction that the fixed point moves.
2. Scale the translated object from step one
3. Back translate the scaled object to its original position



Fig(3.1): a transformation sequence for scaling an object about a specified fixed position point using scaling transformation.

Example 3.3: Scale the rectangle (12,4),(20,4),(12,8),(20,8) with SX=2, SY=2 so the point (12,4) being the fixed point.

Solution:

1- Translate the object with TX= -12 and TY= -4 so the point (12,4) lies on the origin

(12,4) ===> (0,0)
(20,4) ===> (8,0)
(12,8) ===> (0,4)
(20,8) ===> (8,4)

2- Scale the object by SX=2 and SY=2

(0,0) ===> (0 , 0)
(8,0) ===> (16 , 0)
(0,4) ===> (0 , 8)
(8,4) ===> (16 , 8)

3- Back translate the scaled object with TX= 12 and TY= 4

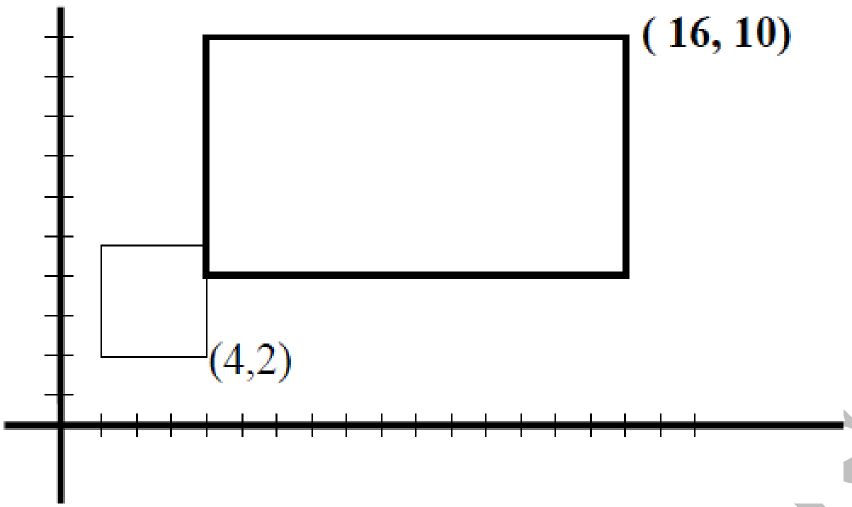
(0 , 0) ===> (12 ,4)
(16 ,0) ===> (28 , 4)
(0 , 8) ===> (12 ,12)
(16 , 8) ===> (28 ,12)

Example 3.4 : Scale the square (1,2) , (4,2) , (1,5) , (4,5) with 4 units in the X-axis ,and 2 units in the Y-axis

Solution:

$$\begin{bmatrix} X1new & Y1new & 1 \\ X2new & Y2new & 1 \\ X3new & Y3new & 1 \\ X4new & Y4new & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 2 & 1 \\ 1 & 5 & 1 \\ 4 & 5 & 1 \end{bmatrix} * \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 1 \\ 16 & 4 & 1 \\ 4 & 10 & 1 \\ 16 & 10 & 1 \end{bmatrix}$$



H/W: Scale the triangle (80,40),(40,80),(120,80) by 0.25

3- Rotation

Another useful transformation is the rotation of an object about a specified pivot point. In the rotation, the object is rotated Ǿ about the origin. The convention is that the direction of rotation is counterclockwise if Ǿ is a positive angle and clockwise if Ǿ is a negative angle.

1-Rotation about the origin

The rotation matrix to rotate an object about the origin in anticlockwise direction is:

Cosθ	Sinθ	0
- Sinθ	Cosθ	0
0	0	1

Alternatively, an equation:

$$X_{new} = X * \text{Cos}\theta - \text{Y} * \text{Sin}\theta$$

$$Y_{new} = Y * \text{Cos}\theta + \text{X} * \text{Sin}\theta$$

When Ǿ=90, the matrix that causes a rotation through an angle of 90 (Π/2) is

0	1	0
- 1	0	0
0	0	1

When Ǿ=180

-1	0	0
0	-1	0
0	0	1

When Ǿ=270

0	-1	0
1	0	0
0	0	1

When Ǿ=360

1	0	0
0	1	0
0	0	1

Example 3.5: rotate the line P1(1,6) and P2(5,1) anticlockwise 90 degrees.

Solution:

1	6	1
5	1	1

*

0	1	0
-1	0	0
0	0	1

=

-6	1	1
-1	5	1

❖ **<u>Rotate about a specific point (XP, YP)</u>**

We need three steps:

1- translate the points (and the object) so that the point (XP, YP) lies on the origin

XP1= X - XP

YP1= Y - YP

2- rotate the translated point (and the translated object) by Ǿ degree about the origin to obtain the new point (XP2, YP2)

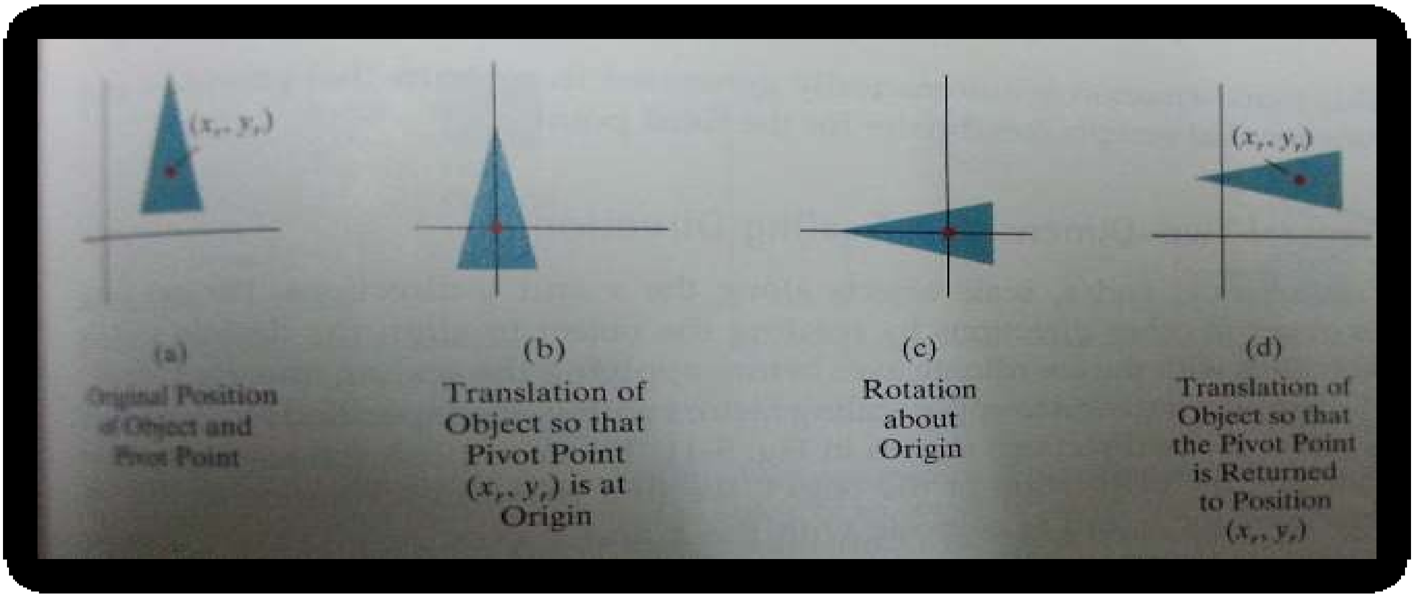
XP2=XP1 * Cos Ǿ - YP1 * Sin Ǿ

YP2=YP1 * Cos Ǿ + XP1 * Sin Ǿ

3- : Back translation

XP3= XP2 + XP

YP3= YP2 + YP



Fig(3.2) : a transformation sequence for rotating an object about a specified point using rotation transformation.

Note: Rotation in clockwise direction:

In order to rotate in clockwise direction we use a negative angle, and because:

Cos (-Ǿ) = Cos Ǿ

Sin (-Ǿ) = - sin Ǿ

Therefore, the matrix will be (for clockwise rotation):

Cos Ǿ	- sin Ǿ	0
sin Ǿ	Cos Ǿ	0
0	0	1

In equations: Xnew=X * Cos Ǿ + Y * Sin Ǿ

Ynew= Y * Cos Ǿ - X * Sin Ǿ

H/W:

1- Rotate the square (3,2),(5,2),(3,4),(5,4) counterclockwise with Ǿ =45 around the point (3,2)

2- : Performa counterclockwise 45 rotation of triangle A (2, 3), B (5,5) and C(4,3) about point (2,3).

3- **Ro**tate the object defined by (43,88), (84,50),(66,72) in rotating at anti-clockwise by angle 53 then scale it with scaling factor Sx=1, Sy=2 where (43,88) is the fixed point?

Hint to the solution of H.W(3)

1- Translate (all the points)by the value of the fixed point to the origin
2- Rotate in anticlockwise by angle 53 degree

3- Scale with values Sx=1, Sy=2

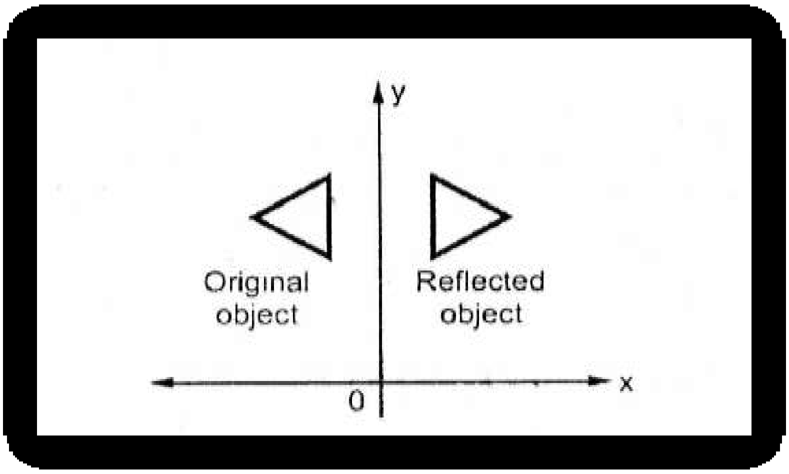
4- Translate the origin back to the fixed point

4- Reflection and Shear Transformations

The three basic transformations of scaling, rotating, and translating are the most useful and most common. There are some other transformations which are useful in certain applications. Two such transformations are reflection and shear.

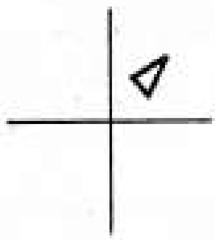
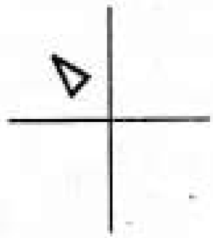
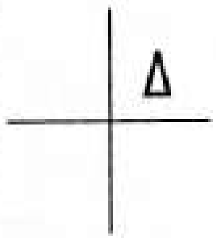
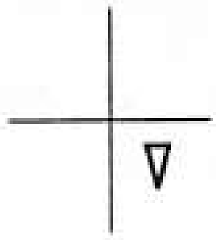
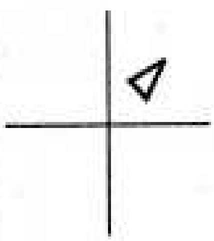
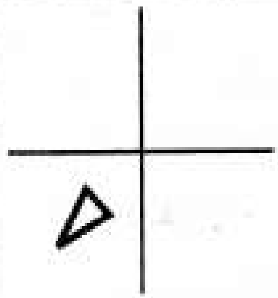
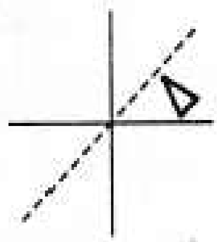
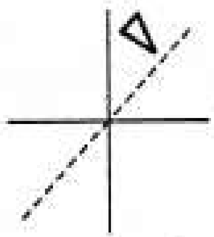
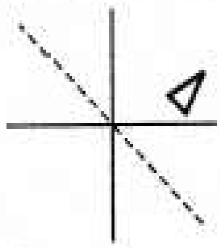
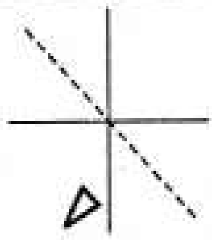
4.1 Reflections

A reflection is a transformation that produces a mirror image of an object relative to an axis of reflection. We can choose an axis of reflection in the x-y plane or perpendicular to the x-y plane. Fig (3.3) gives an example of the reflection in the y-direction.



1 Reflection about y-axis

Table3.1 gives examples of some common reflections.

Reflection	Transformation matrix	Original image	Reflected image
Reflection about Y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		
Reflection about X axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		
Reflection about origin	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		
Reflection about line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		
Reflection about line $y = -x$	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		

1. Reflection on the X-axis

1	0	0
0	-1	0
0	0	1

OR

$X_{new} = X$

$Y_{new} = -Y$

2. Reflection on the Y-axis

-1	0	0
0	1	0
0	0	1

OR

$X_{new} = -X$

$Y_{new} = Y$

3. Reflection on the origin

-1	0	0
0	-1	0
0	0	1

OR

$X_{new} = -X$

$Y_{new} = -Y$

4. Reflection on the line Y=X

0	1	0
1	0	0
0	0	1

OR

$X_{new} = Y$

$Y_{new} = X$

5. Reflection on the line Y=-X

0	-1	0
-1	0	0
0	0	1

OR

$X_{new} = -Y$

$Y_{new} = -X$

Example 3.6: Reflect the point P(3,2) in **A-** X axis; **B-** Y axis; **C-** origin; **D-**line Y=X;

Solution :-

A.

3	2	1

*

1	0	0
0	-1	0
0	0	1

=

3	-2	1

B. -

3	2	1

*

-1	0	0
0	1	0
0	0	1

=

-3	2	1

C. -

3	2	1

*

-1	0	0
0	-1	0
0	0	1

=

-3	-2	1

D-

3	2	1

*

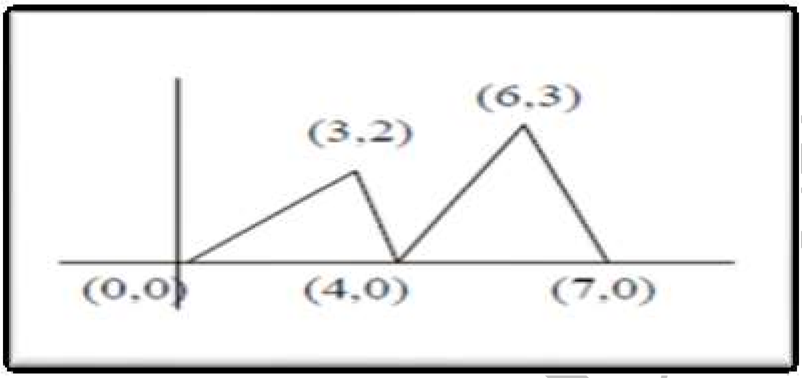
0	1	0
1	0	0
0	0	1

=

2	3	1

H/w

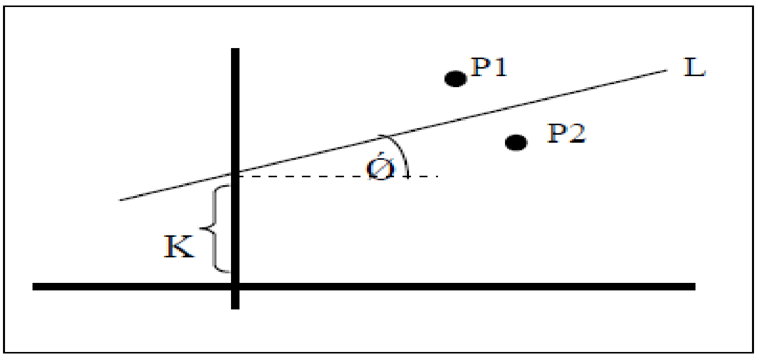
What (3*3) matrix will change the center of the scene to the origin, and reflect the mountains in the lake? [The center of the scene is (4, 0)]



2- **Ro**tate the object defined by (44,68), (104,66),(70,102) in rotating at counter-clockwise by angle 37 degrees after scaling with scale factor Sx=3, Sy=2 where (44,68) is the fixed point?

Reflection on an arbitrary line

To reflect an object on a line that does not pass through the origin, which is the general case:



As shown in the figure, let the line L intercept with Y axis in the point (0, K) and have an angle of inclination Ǿ degree with respect to the positive direction of X-axis. To reflect the point P1 on the line L, we follow the following steps:

1- Move all the points up or down (in the direction of Y-axis) so that L pass through the origin

T=

1	0	0
0	1	0
0	-k	1

2- Rotate all the points through (-Ǿ) degree about the origin making L lie along the X axis

R=

Cosθ	-Sinθ	0
Sinθ	Cosθ	0
0	0	1

3- Reflect the point P1 on the X axis

RefX=

1	0	0
0	-1	0
0	0	1

4- Rotate back the points by (Ǿ) degree so that L back to its original orientation

R_{-1} =

Cosθ	Sinθ	0
- Sinθ	Cosθ	0
0	0	1

5- Shift in the direction of Y-axis so that L is back in its original position

T_{-1} =

1	0	0
0	1	0
0	k	1

The sequence of matrices needed to perform this non-standard reflection is:

$$S=T * R * RefX * R_{-1} * T_{-1}$$

S=

Cos 2Ǿ	Sin 2Ǿ	0
Sin 2Ǿ	- Cos 2Ǿ	0
- K Sin 2Ǿ	K+K Cos 2Ǿ	1

***Example 3**.7*:

Find the single matrix that causes all the points in the plane to be reflected in the line with equation *Y=0.5X+2*, then apply this matrix to reflect the triangle with vertices at A(2,4), B(4,6), C(2,6) in the line.

Solution:

The Cartesian equation of a line in 2D is *Y= M * X +b* where b is the intersection of the lint with the Y-axis and M is a gradient of the line *M= ΔY / ΔX = Tan Ǿ*

So the line ***Y=0.5 X + 2*** has gradient *M= 0.5* and intersect with the Y-axis at the point where *y=2*

So K=2 , Tan Ǿ =0.5 □ Ǿ=26.57

2Ø=53.13 , Cos 2Ø=0.6 , Sin 2Ø=0.8

S=

0.6	0.8	0
0.8	-0.6	0
-1.6	3.2	1

To reflect the triangle on the line:

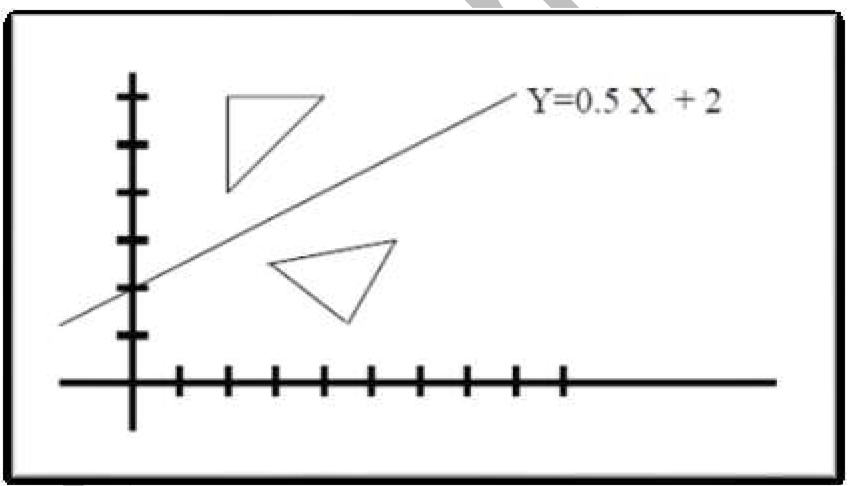
2	4	1
4	6	1
2	6	1

*

0.6	0.8	0
0.8	-0.6	0
-1.6	3.2	1

=

2.8	2.4	1
5.6	2.8	1
4.4	1.2	1



4.2 Shearing

It produces a distortion of an object. There are two types of shearing

1- Y- shearing

It transforms the point (X, Y) to the point (Xnew, Ynew) where

Xnew= X

Ynew= Y+Shy * X where Shy≠0

The matrix is

1	shy	0
0	1	0
0	0	1

Y shearing moves a vertical line up or down depending on the sign of the shear factor Shy. A horizontal line is distorted into a line with slop Shy. And vis versa.

2- X- shearing

It transforms the point (X, Y) to the point (Xnew, Ynew) where

Xnew= X+Shx * Y where Shy≠0

Ynew= Y

The matrix is

1	0	0
shx	1	0
0	0	1

Example 3.6 : Share the object (1,1) , (3,1) , (1,3) , (3,3) with a: Shx=2
b: Shy=2

Solution : a- Shx=2

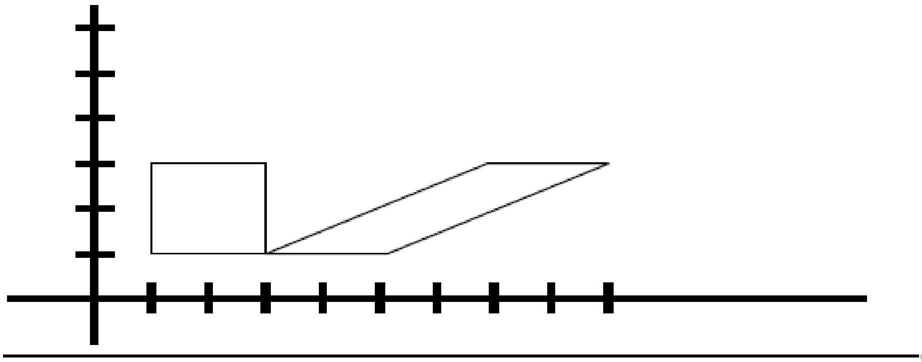
1	1	1
3	1	1
1	3	1
3	3	1

*

1	0	0
2	1	0
0	0	1

=

3	1	1
5	1	1
7	3	1
9	3	1



b- shy

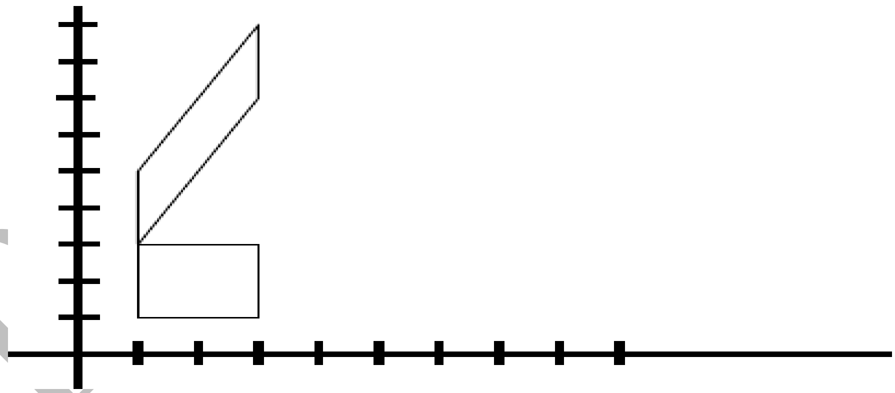
1	1	1
3	1	1
1	3	1
3	3	1

*

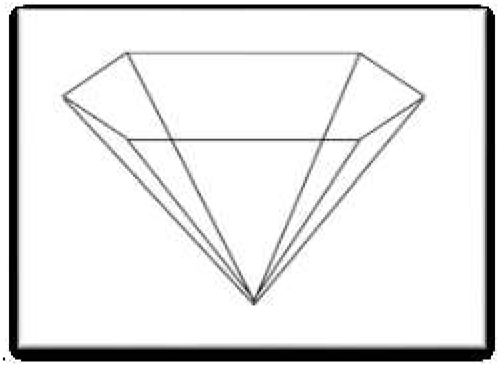
1	2	0
0	1	0
0	0	1

=

1	3	1
3	7	1
1	5	1
3	9	1



H/w: draw the object (5,30),(-5,30),(-11,24),(11,24),(5,18), (-5,18),(0,0)



1- Share the object with Shx=-1
2- The scale on Sx=2 and Sy=1
3- Rotate the object 11 times with Ǿ= Π/6, draw the object after each rotation